

Development of Finite Difference Explicit and Implicit Numerical Reservoir Simulator for Modelling Single Phase Flow in Porous Media

Aphu Elvis Selase^{1,*}, Brantson Eric Thompson², Addo Bright Junior³, Akunda Doreen⁴

¹College of Public Administration, Huazhong University of Science and Technology, Wuhan, China

²School of Energy Resources, China University of Geosciences (Beijing), Beijing, China

³Department of Economics and Geography & Resource Development, University of Ghana, Legon, Ghana

⁴College of Public Administration, Huazhong University of Science and Technology, Wuhan, China

Email address:

elvis.qapito@yahoo.com (A. E. Selase)

*Corresponding author

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Abstract: Every petroleum reservoir requires some means of predicting future performances as well as optimizing recovery of hydrocarbons under various operating conditions. Moreover, there is a need to simulate fluid flow in porous media due to the uncertainty and heterogeneity that is associated with petroleum reservoirs. Therefore, this study developed 1D finite difference explicit and implicit numerical reservoir simulator for modeling single phase flow in porous media. The explicit and implicit simulator developments consist of physical modeling, mathematical modeling, discretization of the models with finite difference scheme and transformation of the models into computer algorithms. Matlab codes were written to describe the fluid flow process to obtain the reservoir pressure distributions for each grid block at each timestep calculation. The explicit formulation linear equation was solved by the direct method while the implicit method was solved by the Jacobi iterative method. The numerical examples graphical plots generated from the simulator illustrate the average reservoir pressure depletion for the finite difference grid blocks. The plots for both the explicit and implicit method indicate decline in average reservoir pressure with time. The explicit and implicit simulators show that the implicit formulation is unconditionally stable than the explicit formulation. This is because the explicit method under certain conditions generates errors in the numerical solutions which tend to go zero during subsequent timestep calculations. Additionally, the porosity sensitivity analyses performed show that the average reservoir pressure decline as the porosity decreases from 30% to 10%. Material balance method was used to determine the average reservoir pressure decline for a one-year production period. The estimated recovery factor at the bubble point pressure is 0.68% of the original oil in place. This low recovery factor is a characteristic of an expansion-drive reservoir which has the least efficient recovery mechanism. Finally, the 1D explicit and implicit finite difference numerical simulators for predicting single phase flow reservoir pressure distributions during production periods are stepping stone towards implementing multiphase fluid flow formulations.

Keywords: Explicit and Implicit Simulators, Material Balance Method, Jacobi Iterative Method, Explicit and Implicit Formulation, Numerical Simulator

1. Introduction

Reservoir simulation is the science of combining physics, mathematics, reservoir engineering, and computer

programming to develop an engineering tool for predicting hydrocarbon reservoir performance under various operating strategies Aziz and Settari, 1979.[1] The classical methods of predicting reservoir performance include analogical, experimental mathematical methods before the advent of

reservoir numerical simulators Zhangxin, 2007 [2]. The analogical methods utilize features of matured reservoirs that are analogous to the new target reservoirs in order to forecast production performance of these reservoirs. The experimental methods measure the physical properties such as saturation, pressure and flow rates in laboratory cores and then scale them up to the whole hydrocarbon accumulation. Additionally, the mathematical methods use model equations to forecast reservoir performance. The comprehensive details about these production forecasting methods can be found in the works of Ahmed 2006, [3] Chen et al. 2007 [4] Gates 2007 [5].

Reservoir simulation is a standard predictive engineering tool in the oil and gas industry used to obtain accurate well performance predictions for a hydrocarbon reservoir under different operating conditions. A hydrocarbon recovery project usually involves a huge capital investment of hundreds of millions of dollars, and the risk associated with its selected development and production strategies must be evaluated and minimized. These risks include such important factors as the complexity of a petroleum reservoir and the fluids filling it, the complexity of hydrocarbon recovery mechanisms, and the applicability of predictive methods. These complexities can be taken into account in reservoir simulation through data input into the simulation model, and this applicability can be estimated through sound engineering practices and accurate reservoir simulation. Therefore, this research involves the development of 1D numerical simulator using explicit and implicit finite difference numerical scheme for predicting pressure distribution in a production reservoir with the following objectives:

- (1) To physically model single-phase fluid flow processes in the reservoir incorporating physics of the underlying physical phenomenon

- (2) To derive a numerical model with the basic properties of both the physical and mathematical models
- (3) To develop computer algorithms (Matlab programming codes) to solve efficiently the systems of linearized algebraic equations arising from the numerical discretization.
- (4) To use the developed simulator to predict average reservoir pressure decline.
- (5) To validate the developed simulator using the material balance method.

This research work is organized into five sections as follows: Section 2 describe the methodology for finite difference formulation. Section 3 describes the results and discussion from the 1D explicit and implicit simulators developed. Finally, section 4 summarizes the conclusions drawn from this study. [6]

2. Methodology for Finite Difference Formulation

The problem to be solved in reservoir modeling is to advance the simulation from the initial conditions to future times. This is accomplished by stepping through the simulation at discrete time intervals called timesteps. Two first order finite difference approximations of forward and backward difference formulations were employed to solve the pressure equations explicitly and implicitly. The discretization involves the process of converting the continuous equations into linear algebraic equations using finite difference method. Figure 1 below shows the discretization steps involved in the development of the numerical simulators.

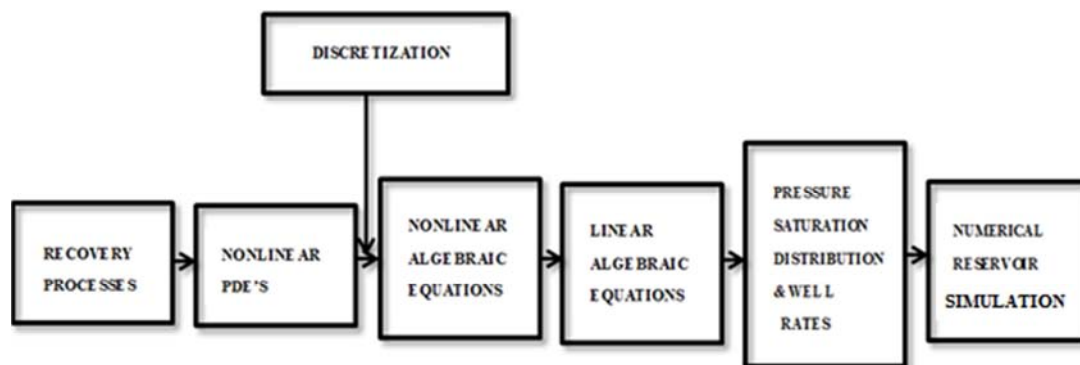


Figure 1. Discretization Steps in the Development of the Numerical Reservoir Simulators Ertekin, 2007 [7].

Some of the advantages of the finite difference method as stated by Abbas Firoozabadi et al 2006 [8] include: simplicity, ease of extension from 1D to 2D and 3D, and also its compatibility with certain aspect of physics of two-and three-phase flow. However, it was pointed out that numerical dispersion and grid dependency are the two major disadvantages of this method. There are basically two types of finite-difference grids used in reservoir simulation: block-centered grid and point-distributed grid. In block-centered

grid, for a rectangular coordinate system, the grid block dimensions are selected first, followed by the placement of points in the central locations of the blocks. The distance between the block boundaries is the defining variable in space. In contrast, the grid points (or nodes) are selected first in the point-distributed grid Abou-Kassem et al., 2006 [9].

2.1. Linear Solvers in Reservoir Simulators

The single facet of a reservoir simulator that has the

greatest combined influence is the technique used to solve the large systems of equations arising from the numerical approximation of the nonlinear fluid flow equations. There are two general methods for solving the matrix equation resulting from the finite difference approximation, namely direct methods and iterative methods.

2.1.1. Direct Methods

These methods transform the original equations into equivalent equations that can be solved more easily. Popular direct solution methods include Gaussian elimination, Gauss-Jordan, and LU (Lower-Upper) decomposition. Ertekin et al. 2007 [7] stated that a direct solver has the capability to produce an exact solution after a fixed number of computations if the computer is able to carry an infinite number of digits. But, because all real computers have a finite word length, the solution obtained through a direct solver will have round-off errors. The direct solver is also limited by its excess computer storage requirements, since it is necessary that both the coefficient matrix and the right-hand vector throughout the solution process are to be stored

2.1.2. Iterative Solution Methods

The iterative process starts with an initial guess of the solution vector, and repeatedly refines the solution until a certain convergence criterion is reached. Popular iterative methods include the Jacobi iteration, Gauss-Siedel, successive over-relaxation (SOR), strongly implicit procedure (SIP), and conjugate-gradient method (CGM). But, in this study, the Jacobi iterative method was employed as the implicit equation solver. The iterative methods are generally slower than their direct counterparts due to the large number of iterations required. Also, the iterative methods do have significant computational advantages especially for large and sparsely populated coefficient matrix. However, Jaan 2006 [10] noted that the iterative methods do not always converge to the desired solution

2.2. Development of the Reservoir Numerical Simulator

The development of the 1D explicit and implicit numerical reservoir simulator include the under listed steps:

- (1) Formulate the partial differential equations (PDE) of the model based on the oil reservoir characteristics, using the three basic laws governing fluid flow in porous media.
- (2) Divide the reservoir into grids and then discretize the formulated partial differential equations in both space and time so as to obtain linearized system of equations.
- (3) Select an appropriate ordering scheme to obtain an

order of coefficients of the linear system of equations and then compute the pressure per grid block of the reservoir.

- (4) Write codes for the system of equations using Matlab programming environment.
- (5) Finally validate the numerical simulator using material balance method

2.3. Mathematical Model

The reservoir characteristics are expressed in term of the set of partial differential equations (PDEs) together with the appropriate initial and boundary conditions that approximate the fluid flow behaviour of the reservoir. There are basically three main laws which govern isothermal reservoir simulation:

- (1) The law of mass conservation
- (2) Darcy's law (transport equation), and
- (3) Equation of state (Phase properties such as density, compressibility and formation volume factor).

2.4. Mathematical Model Description (Basic Assumption)

The mathematical model was developed under the following assumptions:

- 1 Homogeneous reservoir
- 2 Isothermal, single-phase and slightly compressible fluid
- 3 Steady state effective viscosity
- 4 Uniform grid size
- 5 Linear fluid flow into the wellbore
- 6 No-flow reservoir boundary condition
- 7 The fluid is Newtonian
- 8 Capillary and gravity forces are ignored.
- 9 The block-centered grid system was employed
- 10 Chemical reactions are not included.

2.5. Basic Fluid Flow Equation

Considering the mass balance for a control volume, the mass conservation is given as:

(Total mass entering the CV during Δt) – (Total mass leaving the CV during Δt) = (Net change in mass within the CV during Δt).

Since the equation constituting the mathematical model of the reservoir is too complex to be solved by analytical method, therefore, finite difference approximation is used to put the equation in a form that is amenable to solution by digital computer. The process involves spatial and time derivative discretization. The general PDE for a single phase three-dimensional flow through a porous medium may be written in Cartesian coordinates as expressed in Eq. (1) as:

$$\frac{\partial}{\partial x} \left(\beta_c \frac{K_x A_x}{\mu B_o} \frac{\partial P}{\partial x} \right) \Delta x + \frac{\partial}{\partial y} \left(\beta_c \frac{K_y A_y}{\mu B_o} \frac{\partial P}{\partial y} \right) \Delta y + \frac{\partial}{\partial z} \left(\beta_c \frac{K_z A_z}{\mu B_o} \frac{\partial P}{\partial z} \right) \Delta z + q = \frac{V_b}{\alpha_c} \frac{\partial}{\partial t} \left(\frac{\phi}{B_o} \right) \quad (1)$$

From Eq. (1), a linear reservoir system is assumed in the x direction and the discretization is given by Eq. (2) as:

$$\left(\beta_c \frac{A_x k_x}{\mu_1 B_1 \Delta x} \right)_{i+1/2} (p_{i+1} - p_i) - \left(\beta_c \frac{A_x k_x}{\mu_1 B_1 \Delta x} \right)_{i-1/2} (p_i - p_{i-1}) + q_{lsc i} = \left(\frac{V_b \phi c_t}{\alpha_c B_1^0} \right)_i \frac{\partial p_i}{\partial t} \quad (2)$$

Simplifying Eq. (2) by introducing transmissibility terms results into Eq. (3) as:

$$T_{lx_{i+1/2}}(p_{i+1} - p_i) - T_{lx_{i-1/2}}(p_i - p_{i-1}) + q_{lsc_i} = \left(\frac{V_b \phi c_t}{\alpha_c B_l^0} \right)_i \frac{\partial p_i}{\partial t} \quad (3)$$

2.5.1. Forward-Difference Approximations for Fluid Flow Equation

The forward difference approximation for the flow equation at a base time level to the first derivative on the right-hand side of Eq. (3) is expressed in Eq. (4) as:

$$\frac{\partial p_i}{\partial t} \approx \frac{p_i^{n+1} - p_i^n}{\Delta t} \quad (4)$$

Substituting Eq. (4) into Eq. (2) and (3) at time level t^n results into Eq. (5) and (6) as:

$$\left(\beta_c \frac{A_x k_x}{\mu_l B_l \Delta x} \right)_{i+1/2}^n (p_{i+1}^n - p_i^n) - \left(\beta_c \frac{A_x k_x}{\mu_l B_l \Delta x} \right)_{i-1/2}^n (p_i^n - p_{i-1}^n) + q_{lsc_i} = \left(\frac{V_b \phi c_t}{\alpha_c B_l^0 \Delta t} \right)_i (p_i^{n+1} - p_i^n) \quad (5)$$

In terms of transmissibilities,

$$T_{lx_{i+1/2}}(p_{i+1}^n - p_i^n) - T_{lx_{i-1/2}}(p_i^n - p_{i-1}^n) + q_{lsc_i} = \left(\frac{V_b \phi c_t}{\alpha_c B_l^0 \Delta t} \right)_i (p_i^{n+1} - p_i^n) \quad (6)$$

2.5.2. Backward-Difference Approximations for Fluid Flow Equation

The backward-difference approximation generally is used in reservoir simulation because its use does not restrict the size of the timestep for stable solution. The backward difference approximation to the first derivative at the base time level is written in Eqs. (7) and (8) as:

$$\frac{\partial p}{\partial t} = \frac{p(t^{n+1}) - p(t^n)}{\Delta t} \quad (7)$$

$$\frac{\partial p_i}{\partial t} \approx \frac{p_i^{n+1} - p_i^n}{\Delta t} \quad (8)$$

Substituting into Eq. (8) at the time level t^{n+1} results into Eqs. (9) and (10) as:

$$\left(\beta_c \frac{A_x k_x}{\mu_l B_l \Delta x} \right)_{i+1/2}^{n+1} (p_{i+1}^{n+1} - p_i^{n+1}) - \left(\beta_c \frac{A_x k_x}{\mu_l B_l \Delta x} \right)_{i-1/2}^{n+1} (p_i^{n+1} - p_{i-1}^{n+1}) + q_{lsc_i} = \left(\frac{V_b \phi c_t}{\alpha_c B_l^0 \Delta t} \right)_i (p_i^{n+1} - p_i^n) \quad (9)$$

$$T_{lx_{i+1/2}}(p_{i+1}^{n+1} - p_i^{n+1}) - T_{lx_{i-1/2}}(p_i^{n+1} - p_{i-1}^{n+1}) + q_{lsc_i} = \left(\frac{V_b \phi c_t}{\alpha_c B_l^0 \Delta t} \right)_i (p_i^{n+1} - p_i^n) \quad (10)$$

3. Results and Discussion

Matlab is a high-level computer language for scientific computing and data visualization built around an interactive programming environment Ferreira, 2006. [11] The explicit and implicit codes developed in this paper were programmed in Matlab 2016 [12] environment. It is important to acknowledge here that all the numerical examples were taken

from Ertekin et al. 2007. [7]

3.1. Explicit Formulation Calculation Method

The forward difference approximation to the fluid flow equation results in an explicit calculation procedure for the new time level pressures. Solving the forward difference Eq. (6) for the unknown quantity, p_i^{n+1} , yields the expression in Eq. (11) as:

$$p_i^{n+1} = p_i^n + \left(\frac{\alpha_c B_l^0 \Delta t}{V_b \phi c_t} \right)_i q_{lsc_i} + \left(\frac{\alpha_c B_l^0 \Delta t}{V_b \phi c_t} \right)_i \times \left[T_{lx_{i+1/2}}^n p_{i+1}^n \left(T_{lx_{i+1/2}}^n + T_{lx_{i-1/2}}^n \right) p_i^n + T_{lx_{i-1/2}}^n p_{i-1}^n \right] \quad (11)$$

All the terms on the right hand side of Eq. (11) are known because all pressures appearing on this side are at the known (old) time level, n . In this equation, the pressures at the new time level can be obtained explicitly by the use of these known pressures. The Matlab code written for Eq. (11) is given in Appendix A. A graphical user interface (GUI) that accepts input data from the user was designed with Matlab for the explicit formulation finite difference simulator in this

study. The input data required from the user are reservoir and production parameters: (1) Reservoir description data such as overall geometry, grid size specification, permeability and porosity for each gridblock. (2) Fluid PVT properties, such as oil formation volume factor and viscosity, and (3) Specification of well location and production rate. Figure 2 shows the GUI that was created for the explicit simulator.

1D EXPLICIT NUMERICAL SIMULATOR

RESERVOIR DIMENSIONS		FLUID PROPERTIES			
Length, X (ft)	Input 1	Oil Viscosity, μ (cp)	Input 4	Oil FVF @ Bubble point, B_b (RB/STB)	Input 7
Width, Y (ft)	Input 2	Oil Compressibility, C_o (/psia)	Input 5	Bubble point pressure, P_b (psia)	Input 8
Thickness, Z (ft)	Input 3	Initial Oil FVF, B_{oi} (RB/STB)	Input 6	Initial Reservoir Pressure, P_i (psia)	Input 9
PETROPHYSICAL PARAMETERS		PRODUCTION PARAMETERS		NUMBER OF GRIDBLOCKS	
Porosity, Φ (fraction)	Input 10	Oil Production Rate, q (STB/D)	Input 13	Number of Gridblocks	Input 15
Permeability, K_x (Darcy)	Input 11	Production Timestep Size, t (Days)	Input 14	Well Location on Gridblock	Input 16
Water Saturation, S_w (fraction)	Input 12	clear		simulate	

Figure 2. Explicit Numerical Reservoir Simulator Graphical User Interface.

3.2. Example Application of Explicit Scheme with Neumann Boundary Condition

For the 1D block-centered grids shown in Fig. 3 pressure distribution during the first year of production was determined. The initial reservoir pressure is 6000 psia. The rock and fluid properties for this problem are $\Delta x=1000$ ft,

$\Delta y=1000$ ft, $\Delta z=75$ ft, $B_i=1$ RB/STB, $c_i=3.5 \times 10^{-6}$ psi $^{-1}$, $k_x=15$ mD, $\phi=0.18$, $\mu_i=10$ cp, and $B_i^0=1$ RB/STB. Assuming a timestep size of $\Delta t=10$ days. Additionally, assuming that B_i acts as a constant within the pressure range of interest and no flow boundary condition where the flux vanishes everywhere on the boundary.

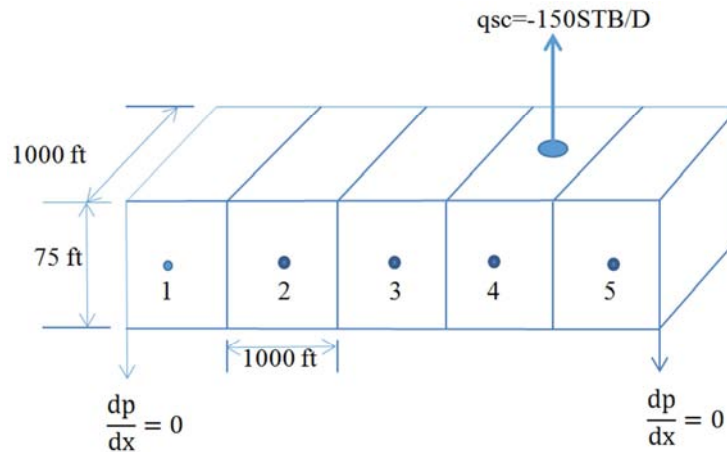


Figure 3. Porous Medium and Block-Centered Grid System Ertekin et al., 2007 [7].

3.3. Explicit Simulation Results

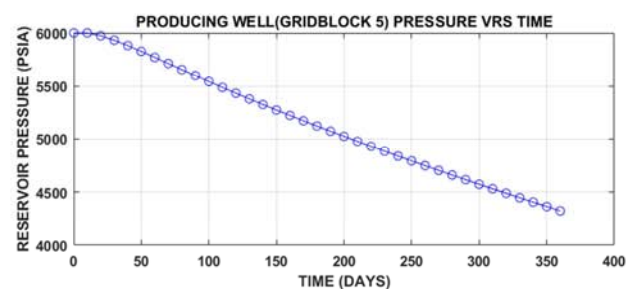
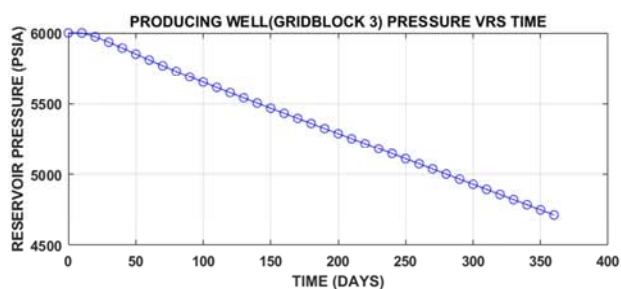
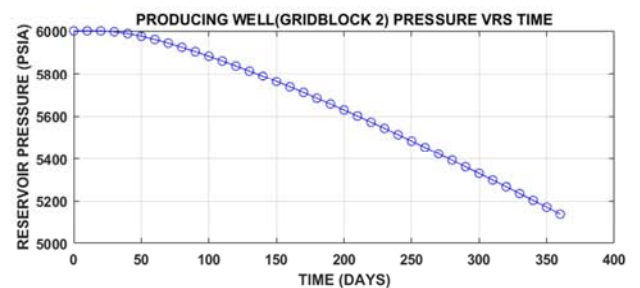
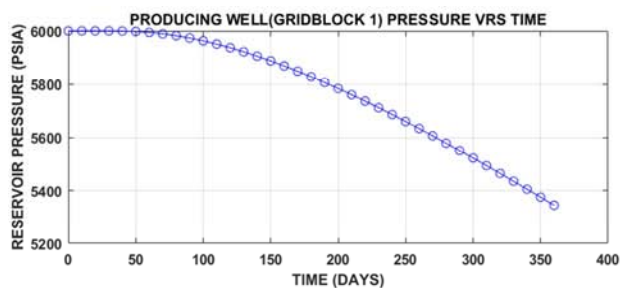
Table 1 and Fig. 4 plots indicate the results that was generated from each timestep after running the simulator.

Table 1. Timestep Results of Explicit Simulator Calculations.

EXPLICIT SIMULATION STUDY RESULTS ($\Delta t=10$ DAYS)					
Time	PRESSURE(Psia)				
(days)	Block 1	Block 2	Block 3	Block 4	Block 5
0	6000.00	6000.00	6000.00	6000.00	6000.00
10	6000.00	6000.00	6000.00	5821.75	6000.00

EXPLICIT SIMULATION STUDY RESULTS ($\Delta t=10$ DAYS)

Time	PRESSURE(PSIA)				
(days)	Block 1	Block 2	Block 3	Block 4	Block 5
20	6000.00	6000.00	5973.14	5697.21	5973.14
30	6000.00	5995.95	5935.61	5602.11	5931.56
40	5999.39	5987.47	5894.45	5523.75	5881.92
50	5997.59	5975.25	5852.61	5455.33	5827.95
60	5994.23	5960.14	5811.22	5393.09	5771.80
70	5989.09	5942.83	5770.66	5334.91	5714.74
80	5982.12	5923.86	5730.94	5279.55	5657.50
90	5973.34	5903.57	5691.99	5226.26	5600.55
100	5962.83	5882.20	5653.70	5174.59	5544.15
110	5950.68	5859.92	5615.93	5124.21	5488.46
120	5937.00	5836.83	5578.60	5074.94	5433.58
130	5921.91	5813.01	5541.62	5026.62	5379.54
140	5905.50	5788.53	5504.91	4979.15	5326.36
150	5887.87	5763.42	5468.42	4932.44	5274.04
160	5869.12	5737.72	5432.11	4886.42	5222.56
170	5849.32	5711.47	5395.93	4841.05	5171.91
180	5828.55	5684.70	5359.87	4796.26	5122.06
190	5806.87	5657.43	5323.89	4752.03	5072.96
200	5784.35	5629.69	5287.98	4708.30	5024.60
210	5761.05	5601.50	5252.12	4665.06	4976.94
220	5737.01	5572.90	5216.30	4622.26	4929.95
230	5712.28	5543.89	5180.52	4579.88	4883.58
240	5686.90	5514.51	5144.77	4537.90	4837.82
250	5660.93	5484.77	5109.04	4496.28	4792.63
260	5634.38	5454.70	5073.32	4455.02	4747.97
270	5607.31	5424.31	5037.62	4414.08	4703.83
280	5579.73	5393.62	5001.93	4373.44	4660.17
290	5551.69	5362.64	4966.25	4333.10	4616.96
300	5523.20	5331.40	4930.57	4293.02	4574.19
310	5494.30	5299.90	4894.90	4253.20	4531.82
320	5465.01	5268.17	4859.23	4213.63	4489.84
330	5435.35	5236.21	4823.57	4174.28	4448.22
340	5405.34	5204.04	4787.91	4135.14	4406.94
350	5375.01	5171.67	4752.25	4096.20	4365.98
360	5344.37	5139.11	4716.59	4057.46	4325.33



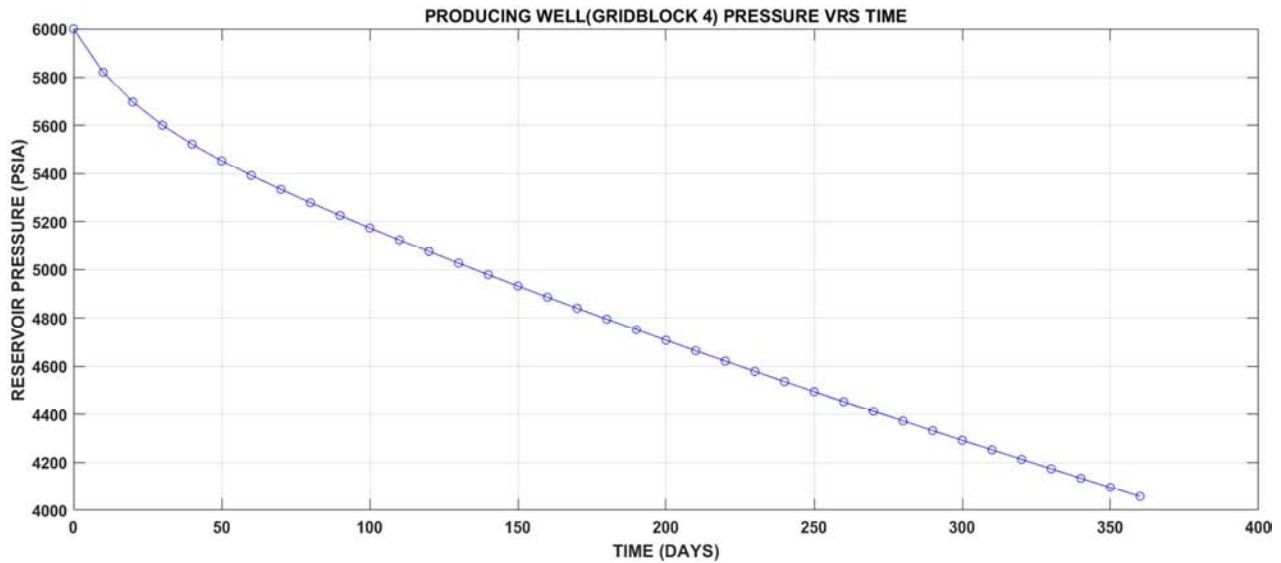


Figure 4. Explicit Pressure against Time Plot for Gridblocks.

3.4. Discussion of Explicit Simulation Results

The first point is that no-flow boundary conditions can be modeled by assigning zero transmissibility at the edges of the porous medium. The second point is that the pressures calculated at identical times are different, depending on the timestep size used in the calculation as shown in Table 1. This is indicative of the approximate nature of the finite difference technique. There is no reason why these approximations should give the same result for different timestep sizes. Because the approximation is first order, the smaller timestep sizes generally give more precise results when compared with the solution of the original PDE. Another point illustrated by this example is that the pressure transient created by fluid withdrawal from the well in Gridblock 4 can only move one cell per timestep as shown in Table 1. This is a property of the explicit method only. For a timestep size of 10 days, it takes the pressure transient 40 days to reach the left boundary, while for a timestep size of 30 days; it would have taken the pressure transient 120 days to reach the boundary. That is, four timesteps are always needed for the pressure transient to reach the left boundary in this example calculation (Table 1). The difference that will occur when a constant pressure (pressure-specified boundary) is specified at either side of the gridblock is that the pressure distribution at the end of the first year of production (360 days) in the gridblock containing the well will be greater than the case of no-flow boundary condition. This is because the pressure that will be specified at either boundary of the gridblock provides support to the gridblock containing the well. This type of boundary condition occurs in a reservoir that is constantly charged by strong water influx, so that the pressure at the interface between the hydrocarbon reservoir and the supporting aquifer remains constant which is known as Dirichlet problem). The method of implementing the specified boundary pressure is the least rigorous method available for block centered grid systems. This is because the

pressure on the boundary is shifted from the gridblock edge to the block center (i.e $\Delta x/2$ away from the reservoir boundary). Although, this method is the least accurate method of implementing boundaries with specified pressure, it is the most practical method of implementation. As a result, it is the most common method of implementing boundaries with specified pressure in reservoir simulators. One method to improve the description of a constant-pressure boundary is to use a more refined grid in the vicinity of the pressure-specified boundary. In general, this method can improve the accuracy of modeling boundaries with specified pressure in block-centered grids. For the explicit solution technique, this may lead to stability problems. This erratic behavior is caused by the conditional stability nature of the explicit forward difference scheme formulation. The forward difference equation is conditionally stable, because under certain conditions, the errors in the solution tend to go to zero during the subsequent timestep calculations. Meanwhile, under other conditions, these same errors propagate uncontrollably during subsequent timestep calculations. This is a serious limitation of the forward difference formulation. In summary, the implementation of the explicit formulation technique involves pressures at the old-time-level only. At this time level, these quantities are known and can be used in an explicit calculation procedure to advance the simulation in time.

3.5. Discussion of Analytical Results

All that is necessary is to devise some means of averaging individual well pressure declines to determine a uniform trend for the reservoir as a whole. The average pressure decline can be determined by the volume weighting of pressures within the drainage area of the well. If p_j and V_j represent the pressure and volume drained by the j^{th} well. Then for an explicit method of production in gridblocks, the average reservoir pressure is given by Eq. (12) as:

$$\bar{P} = \frac{\sum_j p_j V_j}{\sum_j V_j} \quad (12)$$

The obtained average reservoir pressure using Eq. (12) for an explicit numerical computation is given as:

$$\bar{P} = 4846.753806 \text{ psia}$$

Material balance method in Eq. (13) was used to find the total cumulative oil production for a one year production period using the explicit numerical simulator results as:

$$q = \frac{N_p}{t} \quad (13)$$

$$N_p = 150 \times 360$$

$$= 54,000 \text{ STB}$$

Furthermore, the recovery factor at the bubble point pressure is computed using Eq. (14) as:

$$R = \left. \frac{N_p}{N} \right|_{P_b} = \frac{B_{oi}}{B_{ob}} C_t \Delta P \quad (14)$$

$$R = \frac{1}{1} \times 3.5 \times 10^{-6} \times (6000 - 4,057.40)$$

$$R = 0.0067991$$

The estimated recovery factor at the bubble point pressure is 0.0067991 of the original oil in place. This low recovery factor is a characteristic of an expansion-drive reservoir which has the least efficient recovery mechanism Dake, 1998 [13]; Fanchi, 2008 [14]; Tarek, 2006 [15]. A sensitivity study was also conducted on the simulator developed to observe the effect of porosity on average reservoir pressure as shown in Fig. 5. The sensitivity analysis performed to investigate the effect of porosity on average reservoir pressure shows that the average reservoir pressure decline as the porosity decreases from 30% to 10%. This indicates that the time taken to reach the bubble point pressure decreases as the porosity decreases.

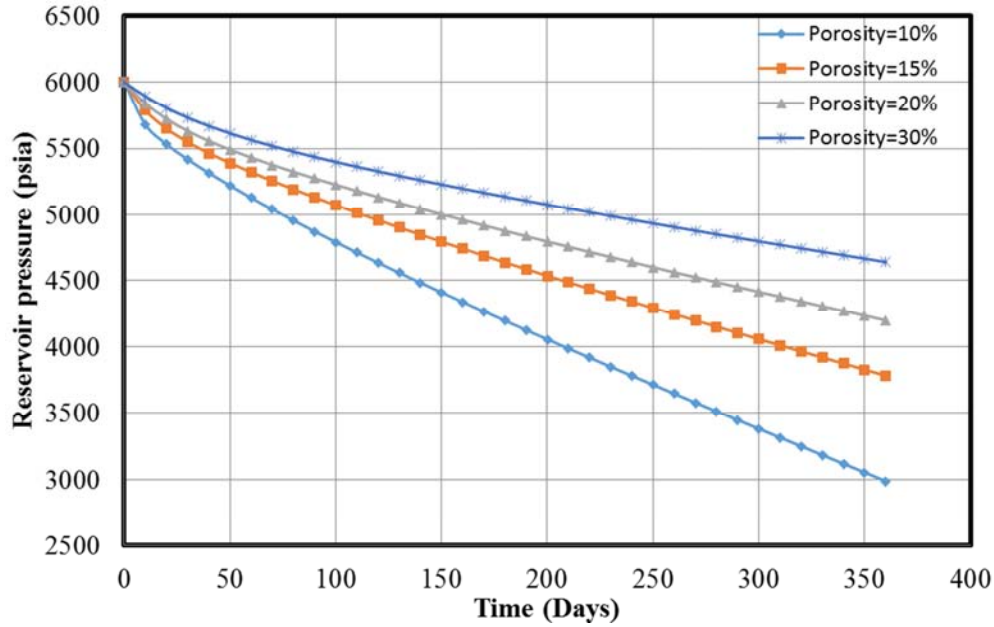


Figure 5. Effect of porosity on average reservoir pressure.

3.6. Implicit Calculation Method

The backward difference approximation to the slightly compressible flow equation results in an implicit calculation procedure for the new-time-level pressures. Rearranging Eq. (10) yields Eq. (15) as:

$$T_{lx_{i+1/2}} p_{i+1}^{n+1} - \left[\left(\frac{V_b \phi c_t}{\alpha_c B_1^0 \Delta t} \right)_i + T_{lx_{i+1/2}} + T_{lx_{i-1/2}} \right] p_i^{n+1} + T_{lx_{i-1/2}} \times p_{i-1}^{n+1} = - \left[q_{lsc_i} + \left(\frac{V_b \phi c_t}{\alpha_c B_1^0 \Delta t} \right)_i p_i^n \right] \quad (15)$$

where the quantities p_{i+1}^{n+1} , p_i^{n+1} and p_{i-1}^{n+1} are all unknowns. Unlike the explicit formulation, Eq. (15) cannot be solved explicitly for p_i^{n+1} because both p_{i+1}^{n+1} and p_{i-1}^{n+1} are also unknown. As a result, Eq. (15) written for all gridblocks and

unknowns must be solved simultaneously or through iterative algorithms. The Matlab code written for Eq. (15) is given in Appendix B. The implicit numerical reservoir simulator interface design is shown in Fig. 6.

Figure 6. Implicit Simulator Graphical User Interface.

3.6.1. Example Application of Neumann Boundary Condition

The same problem described for the explicit method was used for the implicit backward difference formulation. Herein, assuming a timestep size of $\Delta t = 15$ days

3.6.2. Implicit Simulation Results

Table 2. Implicit Simulation Pressure Results.

IMPLICIT SIMULATION STUDY RESULTS ($\Delta t=15$ DAYS)					
Time (days)	PRESSURE(PSIA)				
	Block 1	Block 2	Block 3	Block 4	Block 5
0	6000.00	6000.00	6000.00	6000.00	6000.00
15	5999.08	5995.02	5968.94	5805.44	5964.13
30	5996.29	5983.92	5922.46	5655.36	5907.21
45	5990.90	5967.09	5868.77	5532.90	5838.20
60	5982.51	5945.36	5812.07	5427.97	5762.57
75	5970.91	5919.62	5754.47	5334.45	5683.64
90	5956.11	5890.64	5696.92	5248.60	5603.44
105	5938.21	5859.00	5639.83	5168.10	5523.18
120	5917.37	5825.18	5583.33	5091.48	5443.60
135	5893.80	5789.50	5527.39	5017.78	5365.09
150	5867.70	5752.25	5471.95	4946.39	5287.90
165	5839.29	5713.60	5416.93	4876.86	5212.12
180	5808.77	5673.73	5362.26	4808.89	5137.78
195	5776.32	5632.74	5307.85	4742.26	5064.87
210	5742.11	5590.76	5253.66	4676.81	4993.33
225	5706.30	5547.86	5199.64	4612.39	4923.10
240	5669.03	5504.13	5145.74	4548.90	4854.11
255	5630.42	5459.63	5091.94	4486.24	4786.29
270	5590.60	5414.42	5038.22	4424.34	4719.56
285	5549.67	5368.56	4984.56	4363.13	4653.85
300	5507.71	5322.10	4930.94	4302.55	4589.09
315	5464.83	5275.09	4877.35	4242.54	4525.20
330	5421.09	5227.56	4823.79	4183.06	4462.12
345	5376.56	5179.57	4770.25	4124.06	4399.80
360	5331.32	5131.15	4716.73	4065.50	4338.17

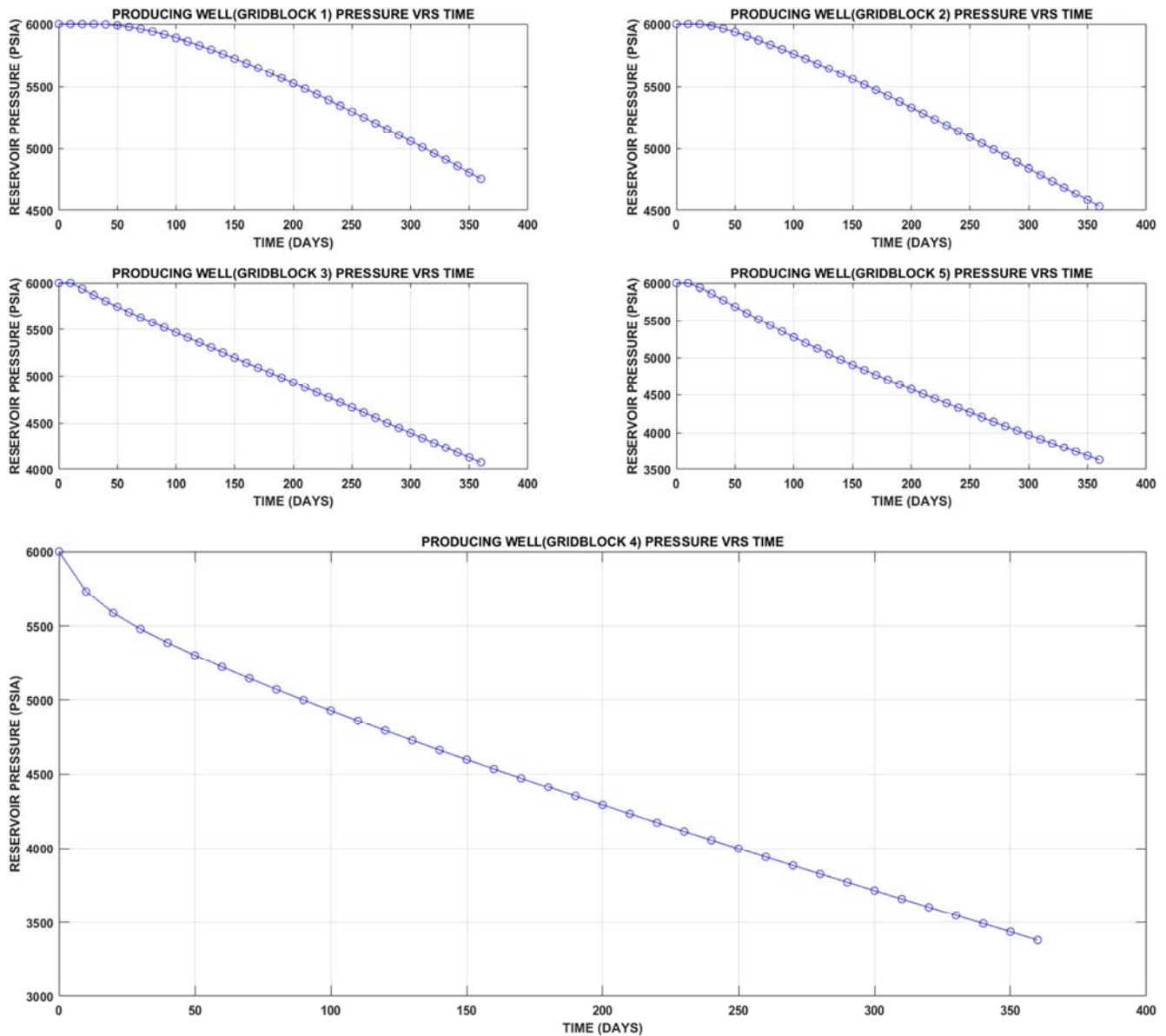


Figure 7. Implicit Pressure against Time Plot for Gridblocks.

3.6.3. Discussion of Implicit Simulation Results

The example calculation illustrates that the amount of computation required for the implicit solution procedure is much greater than that for the explicit solution procedure. This is because, at each timestep, a system of equations must be solved for the unknowns of the problem. In general, most of the computational time used in implicit simulation is in the solution of the linear equation. This example also illustrates that the reservoir pressure can move more than one gridblock per timestep as shown in Table 2. Herein, it has moved to the left boundary during the first timestep. Also, the results from the implicit backward difference formulation and the explicit forward difference formulation will not give identical results even when the same timestep is used. This is because of the approximate nature of the finite difference approach. Moreover, the no-flow boundary can be implemented by assigning zero transmissibility to the boundaries. This

example also demonstrates that no stability problem is associated with the implicit formulation. This is because the implicit formulation is unconditionally stable. This property states that no conditions exist where the backward difference formulation exhibits unstable behavior. Although this statement indicates that any gridblock dimension and /or timestep size can be used with no stability problems, the use of large timestep sizes and block dimensions may result in unrealistic coarse approximations. This becomes apparent in the solution as a departure from the true physics of the problem. However, due to the unconditionally stable nature of the implicit formulation, the backward difference formulation is the most commonly used formulation in petroleum reservoir simulation Ertekin *et al.* 2007. [7]

4. Conclusions

Although, the explicit method is the least accurate method

of implementing boundaries with specified pressure, it is the most practical method of implementation. Thus, it is the most common method of implementing boundaries with specified pressure in numerical reservoir simulators.

On the other hand, the implicit formulation method also demonstrates that no stability problem is encountered as compared to explicit formulation. This is because the implicit formulation is unconditionally stable. Material balance method was also employed to validate the reservoir simulator that was developed. Additionally, the 1D explicit and implicit finite difference numerical simulator developed for predicting single phase pressure distributions in a reservoir during production is a stepping stone towards implementing two or three phase multiphase flow

Numerical methods such as finite element method, integral volume, finite volume method, and variational method could

be employed for the discretization of the PDE governing the fluid flow process and compared to the finite difference method used in this study. This will accommodate both regular and irregular reservoir geometry. In the implicit method implemented in this study, it was observed that the number of iterations required for convergence was large. This may be significantly reduced by introducing preconditioners used to reduce the matrix condition number and speed up the convergence of iterative solvers in order to increase the convergence rate.

Acknowledgements

Thanks to the anonymous reviewers for their comments and suggestions that were instrumental in improving the manuscript.

Appendix

Appendix A: Matlab Code for Explicit Calculation Procedure

```
clear all; close all; clc; clf
%=====
%PROBLEM FORMULATION
%Explicit Finite Difference Formulation
%1D block centered grid
%Determine the pressure distribution during first year of oil production

%=====
%POROUS MEDIUM AND BLOCK CENTERED GRID SYSTEM
%
%
%          ^ qsc=-150STB/D
%          |
%  |-----|-----|-----|-----|
%  |   |   |   |   |   |   |
% 75ft | o | o | o |   | o | o |
%  |   |   |   |   |   |   |
%  | 1 | 2 | 3 | 4 | 5 |
%  |-----|-----|-----|-----|
%  dp/dx=0 1000ft                      dp/dx=0

%=====
%POROUS MEDIUM AND BLOCK CENTERED GRID SYSTEM

%CREATE INPUT VALUES FOR RESERVOIR PROPERTIES
x=input('Enter change in x length value:');
y=input('Enter change in y length value:');
z=input('Enter change in z length value:');
Bo=input('Enter initial formation volume factor:');
Bl=input('Enter liquid phase formation volume factor:');
ct=input('Enter compressibility factor:');
k=input('Enter permeability value:');
ul=input('Enter viscosity value:');
o=input('Enter porosity value:');
t=input('Enter timestep sizes:');
q=input('Enter reservoir flowrate:');
Pi=input('Enter initial pressure');
```

```

%CALCULATE EXPLICIT FORMULAR PROPERTIES
a=y*z; %Calculate the area of gridblocks.unit: ft2
vb=x*y*z; %Calculate the volume of gridblocks.unit: ft3
alpha=(5.615*Bl*t)/(vb*o*ct); %Calculate explicit factor
beta=(1.127*a*k)/(ul*Bl*x); %Calculate the transmissibility of gridblocks
ksi=beta;

%SETTING INITIAL CONDITIONS
P0=0; %For block one Pi-1=P0=1, where i=1
P=ones(1,5)*Pi; %Pi=6000psia for Gridblocks i=1,2,3,4,5

%LOOPING TO CALCULATE TIMESTEP
%tstep=timestep
for tstep=1:5
    for block = 1:5
        %BOUNDARY CONDITIONS FOR GRIDBLOCKS

        alpha(1,block)=(5.615*Bl*t)/(vb*o*ct)
        if block==1
            ksi(1,block)=0
        else
            ksi(1,block)=(1.127*a*k)/(ul*Bl*x)
        end
        if block~=4
            q(1,block)=0
        else
            q(1,block)=-150
        end
        if block==5
            beta(1,block)=0
        else
            beta(1,block)=(1.127*a*k)/(ul*Bl*x)
        end
        gamma=beta+ksi;
        %CALCULATION OF PRESSURE DISTRIBUTION IN BLOCKS AT EVERY TIMESTEP
        if block ~=5
            if block ==1
                P(tstep + 1,block) = P(tstep,block)+ alpha(1,block)*q(1,block) +alpha(1,block)*(beta(1,block) *P(tstep,block+1)- ...
                gamma(1,block)*P(tstep ,block) + ksi(1,block)*P0)
            else
                P(tstep + 1,block) = P(tstep,block)+ alpha(1,block)*q(1,block) +alpha(1,block)*(beta(1,block)*P(tstep,block+1) -
                gamma(1,block)*P(tstep ,block) +ksi(1,block)*P(tstep ,block-1))
            end
        elseif block==5
            P(tstep + 1,block) = P(tstep,block)+ alpha(1,block)*q(1,block) +alpha(1,block)*(beta(1,block)*P(tstep,block) -
            gamma(1,block)*P(tstep ,block) + ksi(1,block)*P(tstep ,block-1))
        end
    end
end

%PLOTTING RESULTS
figure(1)
x=(0:10:360)';
y=P(:,1);
subplot(2,2,1); % defining 1st plotting area
plot(x,y,'bo')
box on;

```

```

title('PRODUCING WELL(GRIDBLOCK 1) PRESSURE VRS TIME');
xlabel('TIME (DAYS)');
ylabel('RESERVOIR PRESSURE (PSIA)');
grid on

x=(0:10:360)';
y=P(:,2);
subplot(2,2,2); % defining 1st plotting area
plot(x,y,'-bo')
box on;
title('PRODUCING WELL(GRIDBLOCK 2) PRESSURE VRS TIME');
xlabel('TIME (DAYS)');
ylabel('RESERVOIR PRESSURE (PSIA)');
grid on

x=(0:10:360)';
y=P(:,3);
subplot(2,2,3); % defining 1st plotting area
plot(x,y,'-bo')
box on;
title('PRODUCING WELL(GRIDBLOCK 3) PRESSURE VRS TIME');
xlabel('TIME (DAYS)');
ylabel('RESERVOIR PRESSURE (PSIA)');
grid on

x=(0:10:360)';
y=P(:,5);
subplot(2,2,4); % defining 1st plotting area
plot(x,y,'-bo')
box on;
title('PRODUCING WELL(GRIDBLOCK 5) PRESSURE VRS TIME');
xlabel('TIME (DAYS)');
ylabel('RESERVOIR PRESSURE (PSIA)');
grid on

figure(2)
x=(0:10:360)';
y=P(:,4);
plot(x,y,'-bo')
box on;
title('PRODUCING WELL(GRIDBLOCK 4) PRESSURE VRS TIME');
xlabel('TIME (DAYS)');
ylabel('RESERVOIR PRESSURE (PSIA)');
grid on

```

Appendix B: Matlab Code for Implicit Calculation Procedure

```
clear all; close all; clc; clf
```

```
=====
%PROBLEM FORMULATION
%Explicit Finite Difference Formulation
%1D block centered grid
%Determine the pressure distribution during first year of oil production
=====
```

```
%POROUS MEDIUM AND BLOCK CENTERED GRID SYSTEM
%
```

```

%               ^ qsc=-150STB/D
%               |
%       |-----|-----|-----|-----|
%       |   |   |   |   |   |   |
% 75ft | o | o | o |   | o | o |
%       |   |   |   |   |   |   |
%       | 1 | 2 | 3 |   | 4 | 5 |
%       |-----|-----|-----|-----|
%       dp/dx=0 1000ft                dp/dx=0

```

```

%POROUS MEDIUM AND BLOCK CENTERED GRID SYSTEM

```

```

%CREATE INPUT VALUES FOR RESERVOIR PROPERTIES

```

```

x=input('Enter change in x length value:');
y=input('Enter change in y length value:');
z=input('Enter change in z length value:');
Bo=input('Enter initial formation volume factor:');
Bl=input('Enter liquid phase formation volume factor:');
ct=input('Enter compressibility factor:');
k=input('Enter permeability value:');
ul=input('Enter viscosity value:');
o=input('Enter porosity value:');
t=input('Enter timestep sizes:');
q=input('Enter reservoir flowrate:');
Pi=input('Enter initial pressure');

```

```

%CALCULATE EXPLICIT FORMULAR PROPERTIES

```

```

a=y*z; %Calculate the area of gridblocks.unit: ft2
vb=x*y*z; %Calculate the volume of gridblocks.unit: ft3
alpha=(5.615*Bl*t)/(vb*o*ct); %Calculate explicit factor
beta=(1.127*a*k)/(ul*Bl*x); %Calculate the transmissibility of gridblocks
ksi=beta;

```

```

%CALCULATION OF PARAMETERS

```

```

Ax=y*z
Vb=x*y*z
T=(beta_c*Ax*kx)/(ul*Bl*x);
rows = 5;
cols = 5;
P = zeros(24,5);
P = ones(1,5)*6000;
Mat_of_Coeff = zeros(rows,cols);
Rhs = zeros(5,1);
coeff = 1;
for block = 1:5
    G(1,block) = (Vb*phi*ct)/(alpha_c*Bl*t)

    if block==1
        ksi(1,block)=0
    else
        ksi(1,block)=0.1268
    end
    if block~=4
        q(1,block)=0
    else
        q(1,block)=-150
    end
end

```



```

if block==5
    beta(1,block)=0
else
    beta(1,block)=0.1268
end
gamma=beta+ksi
if(block == 1)
    Mat_of_Coeff(block, coeff+1) = beta(1,block)
    Mat_of_Coeff(block, coeff) = - G(1,block) - gamma(1,block)
    Mat_of_Coeff(block, coeff+2) = ksi(1,block)
elseif(block ==2)
    Mat_of_Coeff(block, coeff) = beta(1,block)
    Mat_of_Coeff(block, coeff+1) = - G(1,block) - gamma(1,block)
    Mat_of_Coeff(block, coeff+2) = ksi(1,block)
elseif(block>2 && block <5)
    Mat_of_Coeff(block, block-1) = beta(1,block)
    Mat_of_Coeff(block, block) = - G(1,block) - gamma(1,block)
    Mat_of_Coeff(block, block+1) = ksi(1,block)
elseif(block ==5)
    Mat_of_Coeff(block, block-2) = beta(1,block)
    Mat_of_Coeff(block, block) = - G(1,block) - gamma(1,block)
    Mat_of_Coeff(block, block-1) = ksi(1,block)
end
end

for tstep= 1:24
    if(tstep ==1)
        for block = 1:5
            Rhs(block) = -(q(1,block) +Pi*G(1,block))
        end
        P1 = (Mat_of_Coeff\Rhs)'
        for(block =1:5)
            P(tstep+1,block) = P1(block)
        end
    elseif(tstep >=2 && tstep<=24)
        for block = 1:5
            Rhs(block) = -(q(1,block) +P1(block)*G(1,block))
        end
        P1 = (Mat_of_Coeff\Rhs)'
        for(block =1:5)
            P(tstep+1,block) = P1(block)
        end
    end
end

```

JACOBI ITERATIVE SOLUTION METHOD

```

A=Mat_of_Coeff
b= Rhs
% Set initial value of x to zero column vector
x0=zeros(1,5)
% Set Maximum iteration number k_max
k_max=1000;
% Set the convergence control parameter erp
erp=0.0001;
% Show the q matrix
q=diag(diag(A))
% loop for iterations
for k=1:k_max

```

```

for i=1:5
    s=0.0
    for j=1:5
        if j==i
            continue
        else
            s=s+A(i,j)*x0(j)
        end
    end
    x1(i)=(b(i)-s)/A(i,i)
end
if norm(x1-x0)<erp
    break
else
    x0=x1
end
end
% show the final solution
x=x1'
% show the total iteration number
n_iteration=k
    for(block =1:5)
        P(tstep+1,block) = x(block)
    end
end
end
%PLOTING RESULTS
figure(1)
x=(0:15:360)';
y=P(:,1);
subplot(2,2,1); % defining 1st plotting area
plot(x,y,'-bo')
box on;
title('PRODUCING WELL(GRIDBLOCK 1) PRESSURE VRS TIME');
xlabel('TIME (DAYS)');
ylabel('RESERVOIR PRESSURE (PSIA)');
grid on

x=(0:15:360)';
y=P(:,2);
subplot(2,2,2); % defining 1st plotting area
plot(x,y,'-bo')
box on;
title('PRODUCING WELL(GRIDBLOCK 2) PRESSURE VRS TIME');
xlabel('TIME (DAYS)');
ylabel('RESERVOIR PRESSURE (PSIA)');
grid on

x=(0:15:360)';
y=P(:,3);
subplot(2,2,3); % defining 1st plotting area
plot(x,y,'-bo')
box on;
title('PRODUCING WELL(GRIDBLOCK 3) PRESSURE VRS TIME');
xlabel('TIME (DAYS)');
ylabel('RESERVOIR PRESSURE (PSIA)');
grid on

```

```

x=(0:15:360)';
y=P(:,5);
subplot(2,2,4);    % defining 1st plotting area
plot(x,y,'-bo')
box on;
title('PRODUCING WELL(GRIDBLOCK 5) PRESSURE VRS TIME');
xlabel('TIME (DAYS)');
ylabel('RESERVOIR PRESSURE (PSIA)');
grid on

```

```

figure(2)
x=(0:15:360)';
y=P(:,4);
plot(x,y,'-bo')
box on;
title('PRODUCING WELL(GRIDBLOCK 4) PRESSURE VRS TIME');
xlabel('TIME (DAYS)');
ylabel('RESERVOIR PRESSURE (PSIA)');
grid on

```

NOMENCLATURE

A = Cross-sectional area normal in the direction of flow, (ft²)

Bo = Oil formation volume factor (rb/stb)

βC = Unit conversion factor for the permeability coefficient = 1.127,

Δt = Time step (day)

Δx = *Length of a grid block (ft)*

Δy = *Width of a grid block (ft)*

Δz = *Height of a grid block (ft)*

μ = Oil viscosity (cp)

Gi,j,k = Constant part of the transmissibility, (stb/day-psi)

H = Formation thickness (ft)

K = Rock permeability (d)

Kx = Formation permeability in x-direction (d or md)

Ky = Formation permeability in y-direction (d or md)

Kz = Formation permeability in z-direction (d or md)

Lx = Formation length in x-direction (ft)

Ly = Formation length in y-direction (ft)

φ = formation porosity (fraction)

P = Reservoir pressure (psia)

q = Volumetric flow rate of production (stb/D)

qi,j,k = Oil flow rate from well in cell i,j,k (stb/day)

t = Time step (day)

T = Transmissibility, (stb/day-psi)

Ti,j,k+1/2 = Transmissibility along z-direction between cell (i,j,k) and cell (i,j,k+1/2), (stb/d-psi)

Ti,j+1/2,k = Transmissibility along y-direction between cell (i,j,k) and cell (i,j+1/2,k), (stb/d-psi)

Ti+1/2,j,k = Transmissibility along x-direction between cell (i,j,k) and cell (i+1/2,j,k), (stb/d-psi)

Vb = Volume at reservoir condition (rb)

x = x-direction

y = y-direction

z = z-direction

nx = total number of grid cells in the x-direction

ny = total number of grid cells in the y-direction

nz = total number of grid cells in the z-direction

Z = Elevation (ft)

αc = Volume conversion factor (to field unit) = 5.615,

γ = Fluid gravity (psi/ft)

ρ = Density at reservoir condition

o = Oil

n = Previous time step

n+1 = Next time step

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